Student Study Session Topic: Table Problems

The AP Calculus exams include multiple-choice and free-response questions in which the stem of the question includes a table of numerical information from which you are asked questions about the function, its graph, its derivative, or its definite integral. The answers were usually approximations.

Sometimes a function that "modeled" the function in the table was also given and you may be asked similar questions based on the model.

Explanations of what was found were also required in some questions. Thus, starting with a numerical prompt, numerical, graphical, analytic and verbal replies were required.

What you should know how to do:

Here are the most common things that are asked on AP Calculus exam table problems:

1. Approximate a derivative (slope, rate of change, average rate of change) using difference quotients.
2. Use a Riemann sum or a Trapezoidal approximation to approximate a definite integral
3. Explain the meaning of a definite integral in the context of the problem.
4. Calculate a tangent line approximation (local linear approximation).
5. Give the units of the answer (unit analysis).
6. Answer theory questions usually related to the Mean Value Theorem (MVT), Rolle's Theorem, the Intermediate Value Theorem (IVT) or the Extreme Value Theorem (EVT).
7. Give information about the graph of the function.
8. <u>One thing *not* to do</u>: Do not use your graphing calculators to produce a regression equation and use that to answer the questions. Finding regression equations, while very good mathematics, is *not* one of the four things you are allowed to do on the exam with your calculator. Also, a regression model is not the given function, only an approximation of it. You are expected to work from the table using calculus techniques and approximations. Using a regression will not earn any points.

Free-response questions: 2003 AB3, 2002 AB6, 1999 AB3/BC3, 1998 AB3

Student Notes

Multiple-choice selection: 1, 3, 6, 7, 9

a. No calculator: 1 – 3 Graphing calculator allowed 4 – 9

b. Riemann sums questions 1, 4 – 6.

c. Theorems; IVT 7

d. Graph related: 3, 8, 9. Question 3 uses difference quotients and the differences of the difference quotients.

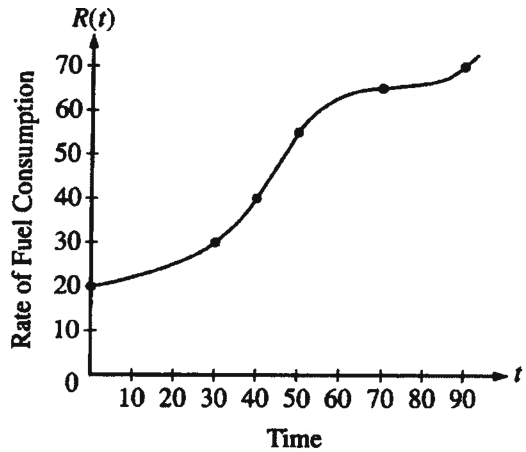
e. Question 2 is included for completeness. The topic is differentiation rules. There is a separate prep session that will cover questions like this.

Answers: 1 C, 2 D, 3 B, 4 B, 5 E, 6 D, 7 A, 8 A, 9 E.

WATCH and LISTEN to the multiple-choice questions being solved

Go to http://tinyurl.com/NMSI-Math-11 **Click on the "Full Screen" arrow. Then click anywhere on the page to see and hear from that point on.**

Click anywhere to go back anytime.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

   (a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

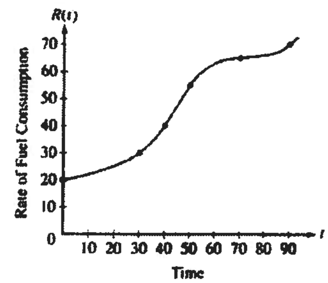
   (c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

   (d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

END OF PART A OF SECTION II

Question 3

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.



t (minutes)	$R(t)$ (gallons per minute)
0	20
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(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

(d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

(a) $R'(45) \approx \dfrac{R(50) - R(40)}{50 - 40} = \dfrac{55 - 40}{10} = 1.5 \text{ gal/min}^2$

2 : $\begin{cases} 1 : \text{a difference quotient using numbers from table and interval that contains 45} \\ 1 : 1.5 \text{ gal/min}^2 \end{cases}$

(b) $R''(45) = 0$ since $R'(t)$ has a maximum at $t = 45$.

2 : $\begin{cases} 1 : R''(45) = 0 \\ 1 : \text{reason} \end{cases}$

(c) $\int_0^{90} R(t)\,dt \approx (30)(20) + (10)(30) + (10)(40) + (20)(55) + (20)(65) = 3700$

Yes, this approximation is less because the graph of R is increasing on the interval.

2 : $\begin{cases} 1 : \text{value of left Riemann sum} \\ 1 : \text{"less" with reason} \end{cases}$

(d) $\int_0^b R(t)\,dt$ is the total amount of fuel in gallons consumed for the first b minutes.

$\frac{1}{b}\int_0^b R(t)\,dt$ is the average value of the rate of fuel consumption in gallons/min during the first b minutes.

3 : $\begin{cases} 2 : \text{meanings} \begin{cases} 1 : \text{meaning of } \int_0^b R(t)\,dt \\ 1 : \text{meaning of } \frac{1}{b}\int_0^b R(t)\,dt \\ <-1> \text{ if no reference to time } b \end{cases} \\ 1 : \text{units in both answers} \end{cases}$

Question 6

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} (3f'(x) + 4)\,dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0. \end{cases}$

The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

(a) $\int_0^{1.5} (3f'(x) + 4)\,dx = 3\int_0^{1.5} f'(x)\,dx + \int_0^{1.5} 4\,dx$ $= 3f(x) + 4x\Big\vert_0^{1.5} = 3(-1 - (-7)) + 4(1.5) = 24$	2: 1 : antiderivative; 1 : answer
(b) $y = 5(x - 1) - 4$ $f(1.2) \approx 5(0.2) - 4 = -3$ The approximation is less than $f(1.2)$ because the graph of f is concave up on the interval $1 < x < 1.2$.	3: 1 : tangent line; 1 : computes y on tangent line at $x = 1.2$; 1 : answer with reason
(c) By the Mean Value Theorem there is a c with $0 < c < 0.5$ such that $f''(c) = \frac{f'(0.5) - f'(0)}{0.5 - 0} = \frac{3 - 0}{0.5} = 6 = r$	2: 1 : reference to MVT for f' (or differentiability of f'); 1 : value of r for interval $0 \le x \le 0.5$
(d) $\lim_{x \to 0^-} g'(x) = \lim_{x \to 0^-} (4x - 1) = -1$ $\lim_{x \to 0^+} g'(x) = \lim_{x \to 0^+} (4x + 1) = +1$ Thus g' is not continuous at $x = 0$, but f' is continuous at $x = 0$, so $f \ne g$. OR $g''(x) = 4$ for all $x \ne 0$, but it was shown in part (c) that $f''(c) = 6$ for some $c \ne 0$, so $f \ne g$.	2: 1 : answers "no" with reference to g' or g''; 1 : correct reason

Name: ________________________ Class: ___________________ Date: __________ ID: A

Table Questions

Multiple Choice
Identify the choice that best completes the statement or answers the question.

No calculator allowed 1 - 3
Calculator allowed 4 - 9

x	2	5	7	8
$f(x)$	10	30	40	20

1\.

The function f is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

a. 110
b. 130
c. 160
d. 190
e. 210

2\.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
−1	6	5	3	−2
1	3	−3	−1	2
3	1	−2	2	3

The table above gives values of f, f', g, and g' at selected values of x. If $h(x) = f(g(x))$, then $h'(1) =$

a. 5
b. 6
c. 9
d. 10
e. 12

3. For all x in the closed interval [2, 5], the function f has a positive fIrst derivative and a negative second derivative. Which of the following could be a table of values for f ?

a.

x	$f(x)$
2	7
3	9
4	12
5	16

b.

x	$f(x)$
2	7
3	11
4	14
5	16

c.

x	$f(x)$
2	16
3	12
4	9
5	7

d.

x	$f(x)$
2	16
3	14
4	11
5	7

e.

x	$f(x)$
2	16
3	13
4	10
5	7

Name: ________________________ **ID: A**

4.

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

A table of values for a continuous function f is shown above. If four equal subintervals of [0, 2] are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)dx$?

a. 8
b. 12
c. 16
d. 24
e. 32

5.

t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at t =0 is 11 feet per second, the approximate value of the velocity at t = 6, computed using a left-hand Riemann sum with three subintervals of equal length, is

a. 26 ft/sec
b. 30 ft/sec
c. 37 ft/sec
d. 39 ft/sec
e. 41 ft/sec

6.

x	2	5	10	14
$f(x)$	12	28	34	30

The function f is continuous on the closed interval [2, 14] and has values as shown in the table above. Using the subintervals [2, 5], [5,10], and [10, 14], what is the approximation of $\int_2^{14} f(x)\,dx$ found by using a right Riemann sum?

a. 296
b. 312
c. 343
d. 374
e. 390

7.

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0, 2] if k=

a. 0
b. $\frac{1}{2}$
c. 1
d. 2
e. 3

8.

x	−4	−3	−2	−1	0	1	2	3	4
$g'(x)$	2	3	0	−3	−2	−1	0	3	2

The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

a. $-2 \le x \le 2$ only
b. $-1 \le x \le 1$ only
c. $x \ge -2$
d. $x \ge 2$ only
e. $x \le -2$ or $x \ge 2$

9.

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

The function f is continuous and differentiable on the closed interval [0,4]. The table above gives selected values of f on this interval. Which of the following statements must be true?

a. The minimum value of f on [0, 4] is 2.
b. The maximum value of f on [0, 4] is 4.
c. $f(x) > 0$ for $0 < x < 4$
d. $f'(x) < 0$ for $2 < x < 4$
e. There exists c, with $0 < c < 4$, for which $f'(c) = 0$.

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.

(a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

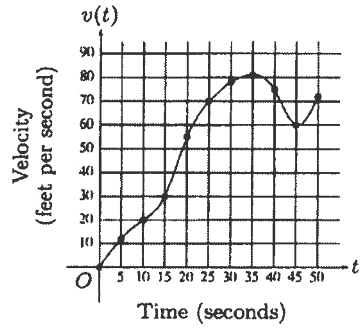
(b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

(c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

Solution	Scoring
(a) $\int_0^{24} R(t)\,dt \approx 6[R(3) + R(9) + R(15) + R(21)]$ $= 6[10.4 + 11.2 + 11.3 + 10.2]$ $= 258.6$ gallons This is an approximation to the total flow in gallons of water from the pipe in the 24-hour period.	3 { 1: $R(3) + R(9) + R(15) + R(21)$ 1: answer 1: explanation
(b) Yes; Since $R(0) = R(24) = 9.6$, the Mean Value Theorem guarantees that there is a t, $0 < t < 24$, such that $R'(t) = 0$.	2 { 1: answer 1: MVT or equivalent
(c) Average rate of flow $\approx$ average value of $Q(t)$ $= \frac{1}{24}\int_0^{24} \frac{1}{79}(768 + 23t - t^2)\,dt$ $= 10.785$ gal/hr or 10.784 gal/hr	3 { 1: limits and average value constant 1: $Q(t)$ as integrand 1: answer
(units) Gallons in part (a) and gallons/hr in part (c), or equivalent.	1: units

1998 Calculus AB Scoring Guidelines



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.
   (a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.
   (b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.
   (c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.
   (d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

(a) Acceleration is positive on $(0, 35)$ and $(45, 50)$ because the velocity $v(t)$ is increasing on $[0, 35]$ and $[45, 50]$

$3\begin{cases} 1: & (0,35) \\ 1: & (45,50) \\ 1: & \text{reason} \end{cases}$

Note: ignore inclusion of endpoints

(b) Avg. Acc. $= \dfrac{v(50) - v(0)}{50 - 0} = \dfrac{72 - 0}{50} = \dfrac{72}{50}$

or 1.44 ft/sec^2

1: answer

(c) Difference quotient; e.g.

$$\frac{v(45) - v(40)}{5} = \frac{60 - 75}{5} = -3 \text{ ft/sec}^2 \quad \text{or}$$

$$\frac{v(40) - v(35)}{5} = \frac{75 - 81}{5} = -\frac{6}{5} \text{ ft/sec}^2 \quad \text{or}$$

$$\frac{v(45) - v(35)}{10} = \frac{60 - 81}{10} = -\frac{21}{10} \text{ ft/sec}^2$$

–or–

Slope of tangent line, e.g.

through $(35, 90)$ and $(40, 75)$: $\dfrac{90 - 75}{35 - 40} = -3$ ft/sec^2

$2\begin{cases} 1: & \text{method} \\ 1: & \text{answer} \end{cases}$

Note: 0/2 if first point not earned

(d) $\displaystyle\int_0^{50} v(t)\,dt$

$\approx 10[v(5) + v(15) + v(25) + v(35) + v(45)]$

$= 10(12 + 30 + 70 + 81 + 60)$

$= 2530$ feet

This integral is the total distance traveled in feet over the time 0 to 50 seconds.

$3\begin{cases} 1: & \text{midpoint Riemann sum} \\ 1: & \text{answer} \\ 1: & \text{meaning of integral} \end{cases}$